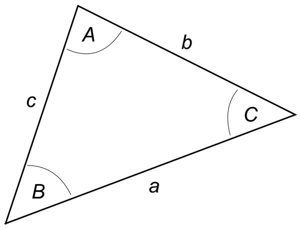
Trigonometry:

For a triangle as shown,



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C};$$

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

For any two angles A and B,

$$\begin{aligned}
\sin A + \sin B &= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right);\\
\sin A - \sin B &= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right);\\
\cos A + \cos B &= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right);\\
\cos A - \cos B &= -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right).
\end{aligned}$$

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A \sin B;\\
\cos(A \pm B) &= \cos A\cos B \mp \sin A \sin B.
\end{aligned}$$

$$\begin{aligned}
2\sin A\cos B &= \sin(A+B) + \sin(A-B);\\
2\cos A\cos B &= \cos(A-B) + \cos(A+B);\\
2\sin A\sin B &= \cos(A-B) - \cos(A+B).
\end{aligned}$$

$$\begin{aligned}
\sin^2 A + \cos^2 A &= 1;\\
1 + \cot^2 A &= \operatorname{cosec}^2 A;\\
1 + \tan^2 A &= \sec^2 A.
\end{aligned}$$

$$\begin{aligned}
\cos 2A &= \cos^2 A - \sin^2 A;\\
\sin 2A &= 2\sin A\cos A;\\
\sin^2 A &= \frac{1-\cos 2A}{2};\\
\cos^2 A &= \frac{1+\cos 2A}{2}.
\end{aligned}$$

Maclaurin series:

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^r}{r!}f^{(r)}(0) + \ldots$$

Taylor series (one variable):

$$\begin{aligned}
f(x) &= f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a)\\
&\quad + \ldots + \frac{(x-a)^r}{r!}f^{(r)}(a) + \ldots
\end{aligned}$$

The binomial theorem:

If n is a positive integer,

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \ldots + x^n.$$

When n is negative or fractional the series is infinite, and converges when $-1 < x < 1$.

Some power series expansions:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \qquad \text{for all } x;$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \qquad \text{for all } x;$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \qquad \text{for all } x;$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \qquad -1 < x \leqslant 1.$$

Green's theorem in the plane:

$$\oint_C (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy.$$

Vector calculus:

If $f(x,y,z)$ is a scalar field and $\mathbf{A}(x,y,z) = A_x\hat{\mathbf{e}}_x + A_y\hat{\mathbf{e}}_y + A_z\hat{\mathbf{e}}_z$ is a vector field,

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2};$$

$$\nabla^2 \mathbf{A} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(A_x\hat{\mathbf{e}}_x + A_y\hat{\mathbf{e}}_y + A_z\hat{\mathbf{e}}_z).$$

If f and g are scalar fields and $\mathbf{A}$ and $\mathbf{B}$ are vector fields,

$$\begin{aligned}
\operatorname{grad}(fg) &= f\operatorname{grad} g + g\operatorname{grad} f;\\
\operatorname{div}(f\mathbf{A}) &= f\operatorname{div}\mathbf{A} + \mathbf{A}\cdot\operatorname{grad} f;\\
\operatorname{curl}(f\mathbf{A}) &= f\operatorname{curl}\mathbf{A} + (\operatorname{grad} f)\times\mathbf{A};\\
\operatorname{curl}\operatorname{grad} f &= 0;\\
\operatorname{div}\operatorname{curl}\mathbf{A} &= 0;\\
\operatorname{curl}\operatorname{curl}\mathbf{A} &= \operatorname{grad}\operatorname{div}\mathbf{A} - \nabla^2\mathbf{A};\\
\operatorname{grad}(\mathbf{A}\cdot\mathbf{B}) &= (\mathbf{B}\cdot\operatorname{grad})\,\mathbf{A} + (\mathbf{A}\cdot\operatorname{grad})\,\mathbf{B}\\
&\quad + \mathbf{B}\times(\operatorname{curl}\mathbf{A}) + \mathbf{A}\times(\operatorname{curl}\mathbf{B});\\
\operatorname{div}(\mathbf{A}\times\mathbf{B}) &= \mathbf{B}\cdot(\operatorname{curl}\mathbf{A}) - \mathbf{A}\cdot(\operatorname{curl}\mathbf{B});\\
\operatorname{curl}(\mathbf{A}\times\mathbf{B}) &= (\mathbf{B}\cdot\operatorname{grad})\,\mathbf{A} - (\mathbf{A}\cdot\operatorname{grad})\,\mathbf{B}\\
&\quad + \mathbf{A}\operatorname{div}\mathbf{B} - \mathbf{B}\operatorname{div}\mathbf{A}.
\end{aligned}$$

Stokes' theorem:

$$\oint_C \mathbf{A}\cdot d\mathbf{l} = \iint_S (\operatorname{curl}\mathbf{A})\cdot d\mathbf{S}.$$

The divergence theorem:

$$\oiint_S \mathbf{A}\cdot d\mathbf{S} = \iiint_V (\operatorname{div}\mathbf{A})\,dV.$$

Spherical polar co-ordinates:

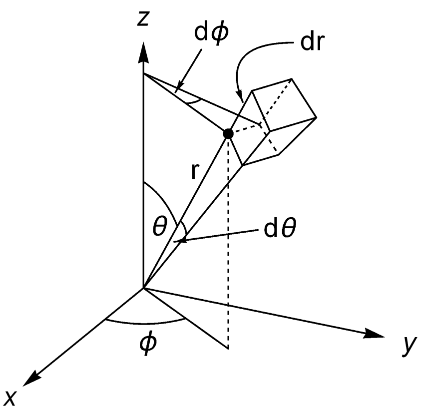


Diagram shows spherical polar co-ordinates (r, θ, ϕ).
Volume element $dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$.

$$\left.\begin{array}{rcl} x &=& r\sin\theta\cos\phi; \\ y &=& r\sin\theta\sin\phi; \\ z &=& r\cos\theta. \end{array}\right\} \quad \begin{array}{l} r \geqslant 0; \\ 0 \leqslant \theta \leqslant \pi; \\ 0 \leqslant \phi < 2\pi. \end{array}$$

If f is a scalar field and
$\mathbf{A} = A_r\hat{\mathbf{e}}_r + A_\theta\hat{\mathbf{e}}_\theta + A_\phi\hat{\mathbf{e}}_\phi$ is a vector field,

$$\begin{aligned}
\operatorname{grad} f &= \frac{\partial f}{\partial r}\hat{\mathbf{e}}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\mathbf{e}}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\mathbf{e}}_\phi; \\
\operatorname{div} \mathbf{A} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right) \\
&\qquad + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}; \\
\operatorname{curl} \mathbf{A} &= \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{\mathbf{e}}_r & r\,\hat{\mathbf{e}}_\theta & r\sin\theta\,\hat{\mathbf{e}}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial\phi} \\ A_r & r\,A_\theta & r\sin\theta\,A_\phi \end{vmatrix}; \\
\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) \\
&\qquad + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}.
\end{aligned}$$

Cylindrical polar co-ordinates:

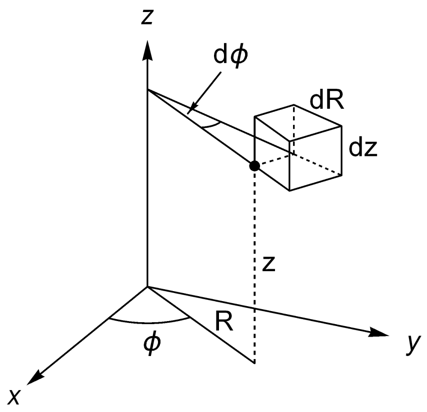


Diagram shows cylindrical polar co-ordinates (R, ϕ, z).
Volume element $dV = R\,dR\,d\phi\,dz$.

$$\left.\begin{array}{rcl} x &=& R\cos\phi; \\ y &=& R\sin\phi; \\ z &=& z. \end{array}\right\} \quad \begin{array}{l} R \geqslant 0; \\ 0 \leqslant \phi < 2\pi; \\ -\infty < z < \infty. \end{array}$$

If f is a scalar field and
$\mathbf{A} = A_R\hat{\mathbf{e}}_R + A_\phi\hat{\mathbf{e}}_\phi + A_z\hat{\mathbf{e}}_z$ is a vector field,

$$\begin{aligned}
\operatorname{grad} f &= \frac{\partial f}{\partial R}\hat{\mathbf{e}}_R + \frac{1}{R}\frac{\partial f}{\partial\phi}\hat{\mathbf{e}}_\phi + \frac{\partial f}{\partial z}\hat{\mathbf{e}}_z; \\
\operatorname{div} \mathbf{A} &= \frac{1}{R}\frac{\partial}{\partial R}\left(R\,A_R\right) + \frac{1}{R}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}; \\
\operatorname{curl} \mathbf{A} &= \frac{1}{R}\begin{vmatrix} \hat{\mathbf{e}}_R & R\,\hat{\mathbf{e}}_\phi & \hat{\mathbf{e}}_z \\ \dfrac{\partial}{\partial R} & \dfrac{\partial}{\partial\phi} & \dfrac{\partial}{\partial z} \\ A_R & R\,A_\phi & A_z \end{vmatrix}; \\
\nabla^2 f &= \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial f}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}.
\end{aligned}$$

Note. The following alternative notations are sometimes used for grad, div, and curl:

$$\begin{aligned}
\operatorname{grad} f &\to \nabla f; \\
\operatorname{div} \mathbf{A} &\to \nabla\cdot\mathbf{A}; \\
\operatorname{curl} \mathbf{A} &\to \nabla\times\mathbf{A}.
\end{aligned}$$

Revised summer 2019 to match the conventions used in the School of Mathematics and Statistics rather than `MathCentre.ac.uk`. This version formatted in LaTeX by CAH; last updated 6.ix.19.

PHYSICS & ASTRONOMY DATA CARD

Physical Constants

Gravitational constant	G	6.673×10^{-11} N m^2 kg^{-2}
Standard acceleration of gravity	g	9.807 m s^{-2}
Speed of light in vacuum	c	2.998×10^{8} m s^{-1}
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity of vacuum	$\varepsilon_0 = 1/\mu_0 c^2$	8.854×10^{-12} F m^{-1}
	$(4\pi\varepsilon_0)^{-1}$	8.988×10^{9} N m^2 C^{-2}
Elementary charge (electron charge = -e)	e	1.602×10^{-19} C
Electron mass	m_e	9.109×10^{-31} kg
Electron charge-to-mass ratio	e/m_e	1.759×10^{11} C kg^{-1}
Proton mass	m_p	1.673×10^{-27} kg
Neutron mass	m_n	1.675×10^{-27} kg
Planck's constant	h	6.626×10^{-34} J s
	$\hbar = h/2\pi$	1.055×10^{-34} J s
Avogadro's constant	N_A	6.022×10^{23} mol^{-1}
Boltzmann's constant	k_B	1.381×10^{-23} J K^{-1}
Molar gas constant	$R = N_\mathrm{A} k_\mathrm{B}$	8.314 J K^{-1} mol^{-1}
Rydberg constant	R_∞	1.097×10^{7} m^{-1}
Bohr radius	a_0	5.292×10^{-11} m
Bohr magneton	$\mu_\mathrm{B} = e\hbar/2m_\mathrm{e}$	9.274×10^{-24} J T^{-1}
Nuclear magneton	$\mu_\mathrm{N} = e\hbar/2m_\mathrm{p}$	5.051×10^{-27} J T^{-1}
Stefan-Boltzmann constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Wien's constant	$b = \lambda_\mathrm{max} T$	2.898×10^{-3} m K
Fine structure constant	$\alpha = e^2/4\pi\varepsilon_0\hbar c$	7.297×10^{-3}
	α^{-1}	137.04

Astronomical constants

Solar mass	$M_\odot$	1.989×10^{30} kg
Solar radius	$R_\odot$	6.960×10^{5} km
Solar luminosity	$L_\odot$	3.90×10^{26} W
Astronomical Unit	AU	1.496×10^{11} m
Light year	LY	9.460×10^{15} m
Parsec	pc	206265 AU $=$ 3.262 LY $= 3.086 \times 10^{16}$ m

Conversion factors and definitions

$k_\mathrm{B}T$ at room temperature	0.0258 eV
Electron volt	1 eV $= 1.602 \times 10^{-19}$ J
Rydberg energy	1 Ry $=$ 13.6 eV
Angstrom	1 Å $= 10^{-10}$ m
Micron	1 μm $= 10^{-6}$ m
Atomic mass unit, $u = m(^{12}\mathrm{C})/12$	1 u = 931.5 MeV/c^2
Standard atmosphere	1 atm $= 1.013 \times 10^{5}$ Pa
Bar	1 bar $= 10^{5}$ Pa

Prefixes

Factor	10^{12}	10^{9}	10^{6}	10^{3}	10^{-3}	10^{-6}	10^{-9}	10^{-12}	10^{-15}	10^{-18}
Prefix	tera	giga	mega	kilo	milli	micro	nano	pico	femto	atto
Symbol	T	G	M	k	m	μ	n	p	f	a